Multi-Scale FEM Modeling of Eddy Currents with a Current Vector Potential *T* in Laminated Media

Karl Hollaus, and Joachim Schöberl
Institute for Analysis and Scientific Computing, TU Wien, A-1040 Vienna, Austria

The aim of the present work is to study multi-scale finite element methods of eddy currents in laminated media in two and three dimensions with a current vector potential. Material properties are assumed to be linear. Hence, approaches are developed for the frequency domain. The weak formulations are derived. Some numerical simulations are presented.

***Index Terms*—Current vector potential, eddy currents, laminated media, multi-scale finite element methods, numerical simulation.**

I. Introduction

An accurate prediction of the eddy current losses in laminated iron cores of electric devices is a challenging task in the design process. Modeling of each laminate requires many finite elements leading to extremely large equation systems. The computational costs to solve these systems are prohibitively high.

The solution obtained by prescribing a current vector potential (CVP) having a single component normal to the lamination [1] or using anisotropic electric conductivity [2] has to be corrected in a post-processing step to consider the effect of the main magnetic flux on the total eddy current losses. These approaches are questionable in the context of nonlinear material properties. Multi-scale finite element methods (MSFEMs) provide the solution in one step taking account of both the main magnetic flux parallel to the lamination and a magnetic stray flux perpendicular to the lamination.

To improve the local approximation the magnetic flux density parallel to the lamination is expanded into orthogonal even polynomials, so-called skin effect sub-basis functions, in [3].

MSFEMs developed in the past utilized a magnetic vector potential $\boldsymbol{A}$, ([4], [5]). In the present work MSFEMs have been developed based on a CVP $\boldsymbol{T}$ describing eddy currents in laminated media in two and three dimensions. A multi-scale order up to order two has been considered.

The developed MSFEM approaches and the weak formulations are presented. Eddy current losses obtained by the new MSFEM have been compared with those obtained by reference solutions of finite element models considering each laminate individually. Numerical simulations are shown.

II. Boundary Value Problem with *T*

The eddy current problem to be solved is sketched in Fig. 1. It consists of a laminated material Ω_m enclosed by air Ω_0, i.e., $\Omega = \Omega_m \cup \Omega_0$ with boundary Γ. The material parameters are the magnetic permeability μ and the electric resistivity ρ.

The CVP $\boldsymbol{T}$ is introduced by

$$\operatorname{curl} \boldsymbol{T} = \boldsymbol{J}, \tag{1}$$

Corresponding author: K. Hollaus (karl.hollaus@tuwien.ac.at).

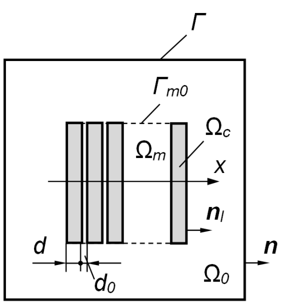

Fig. 1. Eddy current problem model (sketch).

where $\boldsymbol{J}$ stands for the current density [6]. Considering Maxwell's equations for an eddy current problem leads to the boundary value problem

$$\operatorname{curl} \rho \operatorname{curl} \boldsymbol{T} + j\omega\mu \boldsymbol{T} = \boldsymbol{J}_0 \quad \text{in } \Omega \subset \mathbb{R}^3 \tag{2}$$

$$\rho \operatorname{curl} \boldsymbol{T} \times \boldsymbol{n} = \boldsymbol{0} \tag{3}$$

$$\boldsymbol{n} \times \boldsymbol{T} = \boldsymbol{K} \tag{4}$$

in the frequency domain with the imaginary unit j, the angular frequency ω, the impressed current density $\boldsymbol{J}_0$ and the surface current density $\boldsymbol{K}$, respectively.

III. Multi-scale Modeling and Weak formulations

Approaches (6) and (8) are based on the fact that the problem can be observed as a macro-structure with the large dimensions of the iron bulk, on the one hand, and on the other, the micro-structure with the very small thickness of the laminates d and the width of the air gaps d_0 in between (Fig. 1). The mean values u_0 and $\mathbf{T}_0$ consider the large scale variations of the macro-structure and the scalar quantities u_2 and T_{22} and T_{23} and the periodic micro-shape functions ϕ_2, see Fig. 2, and the derivative ϕ_{2x}, respectively, the rough variations of the micro-structure.

The magnetic field strength of the main magnetic field is an even function across the laminates. Therefore, only even terms are considered in the MS approaches (6, 8). The extension to higher order approaches is obviously [5].

Since the solutions of $u_0, \boldsymbol{T}_0, T_{22}$ and wT_{22} are smooth standard finite element basis functions [7] have been used.

A. *Single Component Current Vector Potential T*

The main magnetic flux is parallel to the lamination and perpendicular to the plane of projection [4]. There are basically two possibilities, a single component CVP and a CVP with two components for two-dimensional problems. The later one is of minor practical relevance, thus, not studied here. The single component CVP yields the scalar partial differential equation

$$-\operatorname{div}\rho\operatorname{grad}T + j\omega\mu T = f \tag{5}$$

The higher order multi-scale approach

$$\tilde{T}(x,y) = T_0(x,y) + \phi_2(x)T_2(x,y), \tag{6}$$

see Fig. 2, has been chosen for the complex-valued T, leading to the weak formulation for the finite element method:
Find $(T_{0h}, T_{2h}) \in V_{h,D} := \{(T_{0h}, T_{2h}) : T_{0h} \in \mathcal{U}_h, T_{2h} \in \mathcal{V}_h$ and $T_{0h} = u_D$ on $\Gamma\}$, such that

$$\int_\Omega \rho\nabla\tilde{T}_h\cdot\nabla\tilde{v}_h\,d\Omega + j\omega\int_\Omega \mu\tilde{T}_h\tilde{v}_h\,d\Omega = \int_\Omega f\tilde{v}_h\,d\Omega \tag{7}$$

for all $(v_{0h}, v_{2h}) \in V_{h,0}$, where $\mathcal{U}_h$ is a finite element subspace of $H^1(\Omega)$, $\mathcal{V}_h$ of $L_2(\Omega_m)$ and $\phi_2 \in H^1_{per}(\Omega_m)$.
Index h indicates finite element discretization.

B. *Current Vector Potential* $\boldsymbol{T}$

The two three-dimensional higher order multi-scale approaches

$$\tilde{\boldsymbol{T}} = \boldsymbol{T}_0 + \phi_2\begin{pmatrix}0\\T_{22}\\T_{23}\end{pmatrix} + \left\{0,\quad w_2\begin{pmatrix}\phi_{2x}\\0\\0\end{pmatrix}\right\} \tag{8}$$

have been studied. The weak formulation is:
Find
$(\boldsymbol{T}_{0h}, T_{22h}, T_{23h}, w_{2h}) \in V_{h,K} := \{(\boldsymbol{T}_{0h}, T_{22h}, T_{23h}, w_{2h}) :$
$\boldsymbol{T}_{0h} \in \mathcal{U}_h$, T_{22h} and $T_{23h} \in \mathcal{V}_h$, $w_{2h} \in \mathcal{W}_h$ and
$\boldsymbol{T}_{0h} \times \boldsymbol{n} = \boldsymbol{K}$ on $\Gamma\}$, such that

$$\int_\Omega \rho\operatorname{curl}\tilde{\boldsymbol{T}}_h\cdot\operatorname{curl}\tilde{\boldsymbol{v}}_h\,d\Omega + j\omega\int_\Omega \mu\tilde{\boldsymbol{T}}_h\cdot\tilde{\boldsymbol{v}}_h\,d\Omega = \int_\Omega \boldsymbol{J}_0\cdot\tilde{\boldsymbol{v}}_h\,d\Omega \tag{9}$$

for all $(\boldsymbol{v}_{0h}, v_{22h}, v_{23h}, q_{2h}) \in V_{h,0}$, where $\mathcal{U}_h$ is a finite element subspace of $H(\operatorname{curl},\Omega)$, $\mathcal{V}_h$ of $L_2(\Omega_m)$ and $\mathcal{W}_h$ of $H^1(\Omega_m)$, respectively.

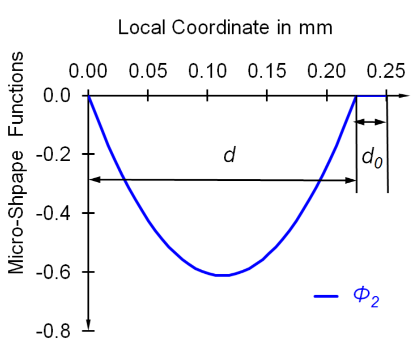

Fig. 2. Micro-shape function ϕ_2.

IV. Numerical Simulations

The iron stack consists of 10 or 100 laminates, see Fig. 1. A thickness of both, iron layer and air gap, of $d + d_0 = 0.25$mm, an unfavorable fill factor of $c_f = 0.9$, a conductivity of $\sigma = 2 \cdot 10^6$S/m and a relative permeability of $\mu_r = 50{,}000$ were selected. Dirichlet boundary conditions were prescribed on Γ to excite the problem. The reference solution obtained by finite element models considering each laminate individually is denoted by RS.

A. *2D problem:*

The problem consists of 100 laminates. The geometry of the problem can be found in [5].

Table 1: Eddy Current Losses in mW/m.

f in Hz	RS	SCVP
50	0.879	0.893
500	18.0	18.5

The results are excellent.

B. *3D problem:*

The problem is a laminated cube with an edge length of 2.5mm and consists of 10 laminates immersed in an homogeneous field. The abbreviations $A1$ and $A2$ stand for the approaches in (8).

Table 2: Eddy Current Losses in pW.

f in Hz	RS	$A1$	$A2$
50	62.9	55.5	55.7
500	531	493	476

In general, such a small example is a demanding task for MSFEM. However, contrary to the case with a magnetic vector potential $\boldsymbol{A}$ presented in [8] the last term in (8) do not account for the edge effects properly. The losses obtained by MSFEM are significantly to small.

Acknowledgment

This work was supported by the Austrian Science Fund (FWF) under Project P 27028-N15.

References

[1] A. Jack and B. Mecrow, "Calculation of three-dimensional electromagnetic fields involving laminar eddy currents," *IEE Proc., Pt. A*, vol. 134, no. 8, pp. 663 –671, september 1987.
[2] K. Preis, O. Bíró, and I. Ticar, "FEM analysis of eddy current losses in nonlinear laminated iron cores," *IEEE Trans. Magn.*, vol. 41, no. 5, pp. 1412–1415, May 2005.
[3] P. Dular, "A time-domain homogenization technique for lamination stacks in dual finite element formulations," *J. Comput. Appl. Math.*, vol. 215, no. 2, pp. 390–399, 2008.
[4] K. Hollaus, A. Hannukainen, and J. Schöberl, "Two-scale homogenization of the nonlinear eddy current problem with FEM," *IEEE Trans. Magn.*, vol. 50, no. 2, pp. 413–416, Feb 2014.
[5] K. Hollaus and J. Schöberl, "A higher order multi-scale FEM with a for 2d eddy current problems in laminated iron," *IEEE Trans. Magn.*, vol. 51, no. 3, 2015.
[6] R. Albanese and G. Rubinacci, "Integral formulation for 3d eddy-current computation using edge elements," *IEE Proc., Pt. A*, vol. 135, no. 7, pp. 457 –462, september 1988.
[7] J. Schöberl and S. Zaglmayr, "High order Nédélec elements with local complete sequence properties," *COMPEL*, vol. 24, no. 2, pp. 374–384, 2005.
[8] K. Hollaus and J. Schöberl, "Multi-scale fem and magnetic vector potential a for 3d eddy currents in laminated iron," ser. 15th Int. IGTE Symp., Graz, Austria, Sep. 2014.